Instructions: Try these problems from the textbook as practice (answers below).

Problems

5-12. Find the limit.

5. $\lim\limits_{(x,y)\to(3,2)} (x^2y^3 - 4y^2)$

7. $\lim\limits_{(x,y)\to(-3,1)} \dfrac{x^2y - xy^3}{x - y + 2}$

9. $\lim\limits_{(x,y)\to(\pi,\frac{\pi}{2})} y\sin(x-y)$

11. $\lim\limits_{(x,y)\to(1,1)} \dfrac{x^2y^3 - x^3y^2}{x^2 - y^2}$

13-18. Show that the limit does not exist.

13. $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^2}{x^2+y^2}$

15. $\lim\limits_{(x,y)\to(0,0)} \dfrac{(x+y)^2}{x^2+y^2}$

17. $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^2\sin^2(x)}{x^4+y^4}$

X. $\lim\limits_{(x,y)\to(0,1)} \dfrac{x(y-1)}{x^2+(y-1)^2}$

19-30. Does the limit exist? If yes, compute it. If no, show that it does not exist.

19. $\lim\limits_{(x,y)\to(-1,-2)} (x^2y - xy^2 + 3)^2$

21. $\lim\limits_{(x,y)\to(2,3)} \dfrac{3x-2y}{4x^2-y^2}$

23. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^2\cos y}{x^2+y^4}$

25. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$

27. $\lim\limits_{(x,y,z)\to(6,1,-2)} \sqrt{x+z}\cos(\pi y)$

29. $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz^2+xz^2}{x^2+y^2+z^4}$

31-34. Compute the limit.

31. $\lim\limits_{(x,y)\to(0,0)} xy\sin\left((x^2+y^2)^{-1}\right)$

33. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^4}{x^4+y^4}$

41-50. Determine the set of points at which the function is continuous.

41. $F(x,y) = \dfrac{xy}{1+e^{x-y}}$

43. $F(x,y) = \dfrac{1+x^2+y^2}{1-x^2-y^2}$

45. $G(x,y) = \sqrt{x} + \sqrt{1-x^2-y^2}$

47. $f(x,y,z) = \arcsin(x^2+y^2+z^2)$

49. $f(x,y) = \begin{cases} 1 & \text{if } (x,y) = (0,0) \\ \dfrac{xy}{x^2+xy+y^2} & \text{otherwise} \end{cases}$

Answers

5. 56 7. -6 9. $\frac{\pi}{2}$ 11. $-\frac{1}{2}$

13. **Hint**: Try $\mathbf{r}_1(t) = \langle 0, t\rangle$ and $\mathbf{r}_2 = \langle t, 0\rangle$ as $t \to 0$.

15. **Hint**: Try $\mathbf{r}_1(t) = \langle t, -t\rangle$ and $\mathbf{r}_2 = \langle t, t\rangle$ as $t \to 0$.

17. **Hint**: Try $\mathbf{r}_1(t) = \langle \arcsin(t), t\rangle$ and $\mathbf{r}_2 = \langle t, 0\rangle$ as $t \to 0$.

X. **Hint**: Try $\mathbf{r}_1(t) = \langle t, t+1\rangle$ and $\mathbf{r}_2 = \langle -t, t+1\rangle$ as $t \to 0$.

19. 25 (continuous)

21. 0 (continuous)

23. Does not exist

25. 2 (multiply by the conjugate)

27. -2 (continuous)

29. Does not exist

31. 0 (squeeze in polar coordinates)

33. 0 (squeeze in polar coordinates)

41-50. Determine the set of points at which the function is continuous.

41. $\mathbb{R}^2$

43. $\{(x, y) : x^2 + y^2 \neq 1\}$

45. $\{(x, y) : 0 < x, x^2 + y^2 < 1\}$

47. $\{(x, y, z) : x^2 + y^2 + z^2 < 1\}$ $f(x, y, z) = \arcsin(x^2 + y^2 + z^2)$

49. $\{(x, y) : x^2 + xy + y^2 \neq 0\}$ (check the limit does not equal 1 at $(0, 0)$)